

## Math 112 homework #2. Due 10/17

*We will not, therefore, perhaps reason illegitimately if we conclude from this that Physics, Astronomy, Medicine, and all the other sciences that have for their end the consideration of composite objects, are indeed of a doubtful character; but that Arithmetic, Geometry, and the other sciences of the same class, which regard merely the simplest and most general objects [...] contain that which is certain and indubitable: for whether I am awake or dreaming, it remains true that two and three make five...*

- Descartes

- (1) Read up to the end of chapter 1 of the course notes, reviewing the material we covered in class.
- (2) Do the following exercises from pages 57-62 of the course notes.  
Take your time, write logically clear and precise solutions, show your work.
  - 1.13
  - 1.14
  - 1.16
  - 1.17 (for each operation that fails to be associative, commutative or have an identity, give an example that displays this failure)
  - 1.24
  - 1.29
  - 1.31 Here  $x^5$  means  $x \cdot x \cdot x \cdot x \cdot x$ .  
(not to hand in, but: why do you think this weird fact is true?)
- (3) Recall the even-odd arithmetic that we defined in class. Let 0 (zero) denote the additive identity in the system. Is it true that for every element  $a$ , we have  $0 \cdot a = 0$ ? Explain.  
(note: here, as in the book, we're using  $\cdot$  to mean multiplication, instead of  $\times$ . While  $\times$  is easier to write on the blackboard, sometimes it can be confused with the letter  $x$ , so the dot is also used.)
- (4) Is it true in even-odd arithmetic that every element has a square root? By this, I mean, is there an element  $a$  such that  $a \cdot a = \text{even}$ , and an element  $b$  such that  $b \cdot b = \text{odd}$ ? Does every number also have a cube root?
- (5) Which elements in one's digit arithmetic have square roots?